A Weihrauch analysis on principles equivalent to Π^1_1-CA_0

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Abstract

Weihrauch reducibility is often used to provide a finer analysis of mathematical theorems that in reverse mathematics are classified among the so called *big five* axiom system (see [Sim09] for a detailed description of the program). Lots of work have been done in finding natural counterparts of the first three axiom systems in the Weihrauch lattice and recently there has been a growing interest in the fourth one (ATR_0). In [KMP18] several candidates have been examined, and the outcome is that there is more than one function in the Weihrauch lattice that is suitable to represent such a system. See [dK19] and [Goh19] for other related results. In this talk we move the attention to the fifth and the "strongest" of this principles, namely Π^1_1-CA_0. Already in [KMP18], it is stated that the natural function representing it in the Weihrauch lattice is the one that maps a countable sequence of trees to the characteristic function of the set of indices corresponding to well-founded trees. Up to our knowledge, the first results at this level are presented in [Hir18], where Hirst shows that the function mentioned above (denoted by $\widehat{\mathsf{WF}}$) is actually equivalent to other functions including PK, that takes as input a tree and outputs its perfect kernel. We will proceed in this direction, studying the relation between different versions of PK and CB, the function corresponding to Cantor-Bendixson Theorem and that maps closed sets to their perfect kernels plus a listing of their isolated points.

References

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