Rough Continuity of Rough Real Functions
Extended Abstract

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Abstract. In the mid 1990s Z. Pawlak relying on the rough set theory initiated the study of rough calculus in his many papers. In the paper, fundamental features of rough continuity of rough real functions are presented in a systematic manner.

In the mid 1990s Z. Pawlak relying on the rough set theory (RST, in short) [1,2,7] initiated the study of rough calculus in many papers, see mainly [5,6,4] [3]. He invented the investigation of its different subfields such as rough continuity–discontinuity, derivatives–integrals, differential equations, etc.

Let I denote a closed interval $I=[0,a]$ ($a\in\mathbb{R}^{\geq 0}$, $a>0$). A *categorization* of I is a sequence $S_I=\{x_i\}_{i\in[n]}\subseteq\mathbb{R}^{\geq 0}$, where $n\geq 1$ and $0=x_0<x_1<\ldots<x_n=a$. S_I is often called the *discretization* of I as well.

Let I_S denote the equivalence relation generated by the categorization S_I which is defined as follows. Let $x,y\in I$. xI_Sy if $x=y=x_i\in S_I$ for some $i\in[n]$, or $x,y\in]x_i,x_{i+1}[$ for some $i\in\{0,1,\ldots,n-1\}$. Hence, the partition I/I_S associated with the equivalence relation I_S is the following:

$$I/I_S=\{\{x_0\},]x_0,x_1[,\{x_1\},\ldots,\{x_{n-1}\},]x_{n-1},x_n[,\{x_n\}\}$$

where $\{x_i\}=[x_i,x_i]$ ($i\in\{0,1,\ldots,n\}$).

The block of the partition I/I_S containing $x\in I$ is denoted by $[\![x]\!]_{I_S}$. In particular, if $x\in S_I$, $[\![x]\!]_{I_S}=\{x\}$. If $x\in[\![x]\!]_{I_S}=]x_i,x_{i+1}[$, $\overline{[\![x]\!]}_{I_S}=[x_i,x_{i+1}]$ is the *closure* of $[\![x]\!]_{I_S}$. Of course, when $x\in S_I$, $[\![x]\!]_{I_S}=\overline{[\![x]\!]}_{I_S}=\{x\}$. Accordingly, any $x\in S_I$ is called *rough isolated point* in I.

In terms of RST terminology, I_S is an indiscernibility relation on I. The members of I/I_S are called *elementary* or *base* sets. Any union of base sets are referred to as *definable* sets. By definition, $\emptyset$ is definable. Their collection is denoted by $\mathcal{D}_{I/I_S}$.

The principal notions of RST are the lower and upper approximation functions, l_S and u_S. Most commonly, their domain and co-domain are the power set of I. In the following, however, the closed intervals of the form $[0,x]$ ($x\in I$) will only be approximated. Therefore,

$$\mathsf{l}_S([0,x])=\cup\{[\![x']\!]_{I_S}\in I/I_S \mid [\![x']\!]_{I_S}\subseteq[0,x]\},$$
$$\mathsf{u}_S([0,x])=\cup\{[\![x']\!]_{I_S}\in I/I_S \mid [\![x']\!]_{I_S}\cap[0,x]\neq\emptyset\}.$$

$\mathsf{PAS}(I) = (I, I/I_S, \mathcal{D}_{I/I_S}, \mathsf{l}_S, \mathsf{u}_S)$ is called *Pawlak approximation space.*

It is said that the number $x \in I$ is *exact* with respect to $\mathsf{PAS}(I)$ if $\mathsf{l}_S([0,1]) = \mathsf{u}_S([0,x])$, otherwise x is *inexact* or *rough* [5]. Of course, $x \in I$ is exact iff $x \in S_I$. Members of I/I_S are called *rough numbers* with respect to $\mathsf{PAS}(I)$.

Let $I = [0, a_I]$ and $J = [0, a_J]$ be two closed intervals with a_I, $a_J \in \mathbb{R}^{\geq 0}$, $a_I, a_J > 0$. Let S_I and P_J be the categorizations of I and J, where $S_I = \{x_i\}_{i \in [n]}$, $P_J = \{y_j\}_{j \in [m]} \subseteq \mathbb{R}^+$ with $m, n \geq 1$, $0 = x_0 < x_1 < \cdots < x_n = a_I$ and $0 = y_0 < y_1 < \cdots < y_m = a_J$.

The corresponding Pawlak approximation spaces are $\mathsf{PAS}(I)$ and $\mathsf{PAS}(J)$. They form a so–called (S_I, P_J)–coordinates system. Real functions $f : I \to J$ are treated in such a coordinate system, and so, they are called rough real functions.

First, the representation of rough functions will be shown. It can be performed by pointwise, block by block, finite sequence, and discrete sequence manner. They rely on lower and upper approximations of rough functions.

The main notion is the pointwise continuity of rough real functions [5]. A function $f : I \to J$ is *roughly continuous* at x if $f(\overline{[x]}_{I_S}) \subseteq \overline{[f(x)]}_{J_P}$, where $\overline{S}$ is the closure of set S. Otherwise, f is *roughly discontinuous* at $x \in I$.

The main part of the paper is different generalizations of rough continuity with respect to block by block, finite sequence, and discrete sequence representations, and the presentation of their mutual connections.

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