▸ BRUNO DINIS, *Quantitative information on the strong convergence of algorithms via proof mining.*
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Proof mining is a program that makes use of tools from mathematical logic in order to analyse mathematical proofs. This analysis is developed with the purpose of extracting quantitative information from proofs, for example in the form of effective bounds and/or algorithms. The main tools from logic are proof interpretations, introduced by Kurt Gödel in the 1950's in order to obtain consistency proofs for theories of arithmetic. The success of the proof mining program is due to the ability of extracting computational content from non-constructive proofs and to the fact that from such analysis it is usually possible to improve the results analysed by means of weakening the hypotheses necessary to prove them, thus leading to more general results. Moreover, in the improved results the logical tools used to analyse the original proof are not visible and can therefore be read by non-logicians.

In this talk I will report on joint work with P. Pinto [2, 3, 4] concerning recent quantitative results, obtained via the analysis of several results concerning the strong convergence of different algorithms. Namely, a generalized proximal point algorithm which involves multiple parameters and modified versions of the Krasnosel'skiĭ-Mann, the forward-backward and the Douglas-Rachford algorithms with Tikhonov regularization terms.

I will start by addressing a theorem by Yao and Noor [6, theorem 3.3] and a theorem by Wang and Cui [7, theorem 1] which prove, under different assumptions, the strong convergence of the multi-parametric proximal point algorithm to the nearest projection point onto the set of zeros of the operator. The quantitative analysis provides explicit bounds on the metastability (in the sense of Terence Tao) for the convergence and information on the asymptotic regularity of the iteration.

I will then consider the strongly convergent modified versions of the algorithms with Tikhonov regularization terms, introduced by Boţ, Csetnek and Meier in [1]. The quantitative information extracted consists of rates of asymptotic regularity and metastability.

The arguments used in the quantitative versions avoid the use of sequential weak compactness and use only a weak form of the projection argument, as explained in [5]. Moreover, some of the results are made possible by an arithmetization of the lim sup.

[1] R. Boţ, E. Csetnek and D. Meier, *Inducing strong convergence into the asymptotic behaviour of proximal splitting algorithms in Hilbert spaces*, ***Optim. Methods Softw.*** 34(3), 489–514 (2019).

[2] B. Dinis and P. Pinto, *Metastability of the proximal point algorithm with multi-parameters*, (submitted) arXiv:1906.09129v2.

[3] B. Dinis and P. Pinto, *Quantitative results on the multi-parameters proximal point algorithm*, (To appear in Journal of Convex Analysis).

[4] B. Dinis and P. Pinto, *On the convergence of algorithms with Tikhonov regularization terms*, (submitted) arXiv:2005.07149.

[5] F. Ferreira, L. Leuştean and P. Pinto, *On the removal of weak compactness arguments in proof mining*, ***Advances in Mathematics***, 354:106728 (2019).

[6] Y. Yao and M. Noor, *On convergence criteria of generalized proximal point algorithms*, ***Journal of Computational and Applied Mathematics***, 217(1):46 – 55 (2008).

[7] F. Wang and H. Cui, *On the contraction-proximal point algorithms with multi-parameters*, ***Journal of Global Optimization***, 54(3):485–491 (2012).