Choice principles via the existential completion

M.E. Maietti[†], D. Trotta[⋆], and M. Zorzi[⋆]

[†]University of Padova, Italy

[⋆]University of Verona, Italy

Abstract

It is well know that choice principles, such as Zermelo's axiom of choice, are not generally valid in constructive foundations for mathematics which include quotient sets. Also Hilbert's epsilon operator is not acceptable in a constructive setting.

However, some forms of choice principles are relevant in the context of program extraction from constructive proofs, at least in a form of a rule admissible in the chosen foundation or as an axiom to be validated in a realizability model of the constructive foundation as required in [6]. Actually the different "status" of choice principles in the various constructive foundations had even motivated the definition of the two-level foundation for constructive mathematics in [5] and the introduction of the Minimalist Foundation in [4].

In this work we employ tools from categorical logic such as the notion of existential completion of Lawvere's doctrines introduced in [1] to provide an *algebraic characterization of some choice principles including Hilbert's epsilon operator*.

The ultimate goal is to use them to characterize fragments of constructive foundations where they are valid, including the Minimalist Foundation which does not validate even the unique choice rule (see [5]).

To this purpose we also employ the categorical tools of *tripos-to-topos* construction [7] in terms of *elementary existential doctrines* [3] and its connections with the notion of *exact completions* [2].

References

[1] D.Trotta (2019). Existential completion. submitted.

[2] Maietti, M., Pasquali, F., and Rosolini, G. (2017). Triposes, exact completions, and hilbert's ε-operator. *Tbilisi Mathematica journal*, 10(3):141–166.

[3] Maietti, M. and Rosolini, G. (2013). Unifying exact completions. *Appl. Categ. Structures*, 23:43–52.

[4] Maietti, M. E. (2009). A minimalist two-level foundation for constructive mathematics. *Ann. Pure Appl. Log.*, 160(3):319–354.

[5] Maietti, M. E. (2017). On choice rules in dependent type theory. In Gopal, T. V., Jäger, G., and Steila, S., editors, *Theory and Applications of Models of Computation - 14th Annual Conference, TAMC 2017, Bern, Switzerland, April 20-22, 2017, Proceedings*, volume 10185 of *Lecture Notes in Computer Science*, pages 12–23.

[6] Maietti, M. E. and Sambin, G. (2005). Toward a minimalistic foundation for constructive mathematics. In Crosilla, L. and Schuster, P. M., editors, *From sets and types to topology and analysis - Towards practicable foundations for constructive mathematics*, volume 48 of *Oxford logic guides*. Oxford University Press.

[7] Pitts, A. M. (2002). Tripos theory in retrospect. *Math. Struct. in Comp. Science*, 12:265–279.