Rigorous Function Calculi-III

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Abstract

Almost all problems in applied mathematics, including the solution of various classes of differential equation, deal with spaces of real-valued functions on Euclidean domains in their formulation and solution. In this talk, I will describe the results of a programme (Work Package 9 of CID) for developing implementations of various function spaces within the framework of computable analysis, and their implementation in the C++ package Ariadne [1, 2], together with some applications.

I will first recall previous work on defining interfaces for the function types considered, covering (piecewise-)continuous, differentiable, analytic and elementary functions, measurable and integrable functions, and set-valued functions, including the C++ class definitions in Ariadne. Here, I will also highlight some remaining theoretical advances including the Weierstrass appproximation theorem providing the effective equivalence of functions defined by (interval) evaluation with polynomial approximations, the extension of measurable functions to non-metric spaces via *lower-measurable* sets.

Next, I will give details of concrete implementations of the function calculus, including symbolic formulae, piecewise-constant, polynomial and Fourier approximations, and examples of low-level usage of the calculus to solve simple mathematical problems.

Finally, I will illustrate the use of the calculus with examples from dynamical systems, including the use of multivalued functions to solve differential inclusions [3] (with S. Zivanovic-Gonzalez and L. Geretti) and on interpolated functions and Fourier bases and to solve partial differential equations (with S. Selivanova, F. Steinberg and N. Zhong).

References

[1] Pieter Collins, Alberto Casagrande, Luca Geretti, Sanja Zivanovic-Gonzalez, Tiziano Villa, and et al. Ariadne: A C++ library for formal verification of cyber-physical systems, 2003-2020. `www.ariadne-cps.org`.

[2] Pieter Collins, Davide Bresolin, Luca Geretti, and Tiziano Villa. Computing the evolution of hybrid systems using rigorous function calculus. In *Proceedings of the 4th IFAC Conference on Analysis and Design of Hybrid Systems*, 2012.

[3] L. Geretti, D. Bresolin, P. Collins, S. Zivanovic Gonzalez, and T Villa. Ongoing work on automated verification of noisy nonlinear systems with Ariadne. In *Proc. of the 29th International Conference on Testing Software and Systems (ICTSS)*, pages 313–319, 2017.